

Part 1: Multiple Choice. Circle the letter corresponding to the best answer.

1. A city planner is comparing traffic patterns at two different intersections. He randomly selects 12 times between 6 am and 10 pm, and he and his assistant count the number of cars passing through each intersection during the 10-minute interval that begins at that time. He plans to test the hypothesis that the mean difference in the number of cars passing through the two intersections during each of those 12 times intervals is 0. Which of the following is the appropriate test of the city planner's hypothesis?
- (a) Two-proportion z -test
 - (b) Two-sample z -test
 - (c) Matched pairs t -test
 - (d) Two proportion t -test
 - (e) Two-sample t -test

Use the following for questions 2 and 3

Different varieties of fruits and vegetables have different amounts of nutrients. These differences are important when these products are used to make baby food. We wish to compare the carbohydrate content of two varieties of peaches. The data were analyzed with MINITAB, and the following output was obtained:

	N	Mean	StDev	SE Mean
Variety 1	5	33.6	3.781	1.691
Variety 2	7	25.0	10.392	3.927

Difference = μ (Variety 1) - μ (Variety 2)

Estimate for difference: 8.6

T-Test of difference = 0 (vs not =): T-Value = 2.011 P-Value = 0.0791 DF = 8

2. We wish to test if the two varieties are significantly different in their mean carbohydrate content. Which of the following are the appropriate null and alternative hypotheses for this situation?
- (a) $H_0: \mu_1 = \mu_2; H_a: \mu_1 < \mu_2$
 - (b) $H_0: \mu_1 = \mu_2; H_a: \mu_1 > \mu_2$
 - (c) $H_0: \mu_1 = \mu_2; H_a: \mu_1 \neq \mu_2$
 - (d) $H_0: \bar{x}_1 = \bar{x}_2; H_a: \bar{x}_1 < \bar{x}_2$
 - (e) $H_0: \bar{x}_1 = \bar{x}_2; H_a: \bar{x}_1 \neq \bar{x}_2$
3. Assuming the conditions for inference were met, which of the following is the most appropriate conclusion to draw at the $\alpha = 0.05$ level?
- (a) The test provides convincing evidence that the carbohydrate content of variety 1 is higher than variety 2.
 - (b) The test provides convincing evidence that the carbohydrate contents of the two varieties are equal.
 - (c) We accept H_0 : variety 1 has a higher carbohydrate content than variety 2.
 - (d) We reject H_0 : variety 1 has a higher carbohydrate content than variety 2.
 - (e) We cannot reject H_0 : we do not have convincing evidence that the carbohydrate contents are different.

4. Popular wisdom is that eating presweetened cereal tends to increase the number of dental caries (cavities) in children. A sample of children was (with parental consent) entered into a study and followed for several years. Each child was classified as a sweetened-cereal lover or a unsweetened cereal lover. At the end of the study, the amount of tooth damage was measured. Here are the summary data:

Cereal preference		<i>n</i>	Mean	Std. Dev.
	Sweetened	10	6.41	5.0
	Unsweetened	15	5.20	15.0

Assuming the necessary conditions for inference are met, which of the following is an approximate 95% confidence interval for the difference in the mean tooth damage?

- (a) $(6.41 - 5.20) \pm 2.262 \sqrt{\frac{5}{10} + \frac{15}{15}}$
 (b) $(6.41 - 5.20) \pm 2.262 \sqrt{\frac{25}{10} + \frac{225}{15}}$
 (c) $(6.41 - 5.20) \pm 2.145 \sqrt{\frac{25}{10} + \frac{225}{15}}$
 (d) $(6.41 - 5.20) \pm 1.96 \sqrt{\frac{25}{10} + \frac{225}{15}}$
 (e) $(6.41 - 5.20) \pm 1.96 \sqrt{\frac{25}{100} + \frac{225}{225}}$

5. You are constructing a 90% confidence interval for the difference of means from simple random samples from two independent populations. The sample sizes are $n_1 = 6$ and $n_2 = 14$. You draw dot plots of the samples to check the normality condition for two-sample t -procedures. Which of the following descriptions of those dot plots would suggest that it is safe to use t -procedures?

- I. The dot plot of sample 1 is roughly symmetric, while the dot plot of sample 2 is moderately skewed left. There are no outliers.
 II. Both dot plots are roughly symmetric. Sample 2 has an outlier.
 III. Both dot plots are strongly skewed to the right. There are no outliers.

- (a) I only
 (b) II only
 (c) I and II
 (d) I, II, and III
 (e) t -procedures are not recommended in any of these cases.

Use the following for questions 6 – 8:

Janice and her cousin Linda are a little competitive about the relative merits of their home towns. One contest they had was to determine who had more rainy days. They found weather records on the internet and each of them randomly selected 60 days from the past 5 years. Janice found that there had been measurable rainfall on 17 of the 60 days she selected for Asheville, and Linda found that there had been measurable rainfall on 12 of the 60 days she selected for Lincoln. They intend to perform a test of significance on their data, using the hypotheses

$H_0: p_A - p_L = 0$ versus $H_a: p_A - p_L \neq 0$ and the 0.05 significance level.

6. When calculating the test statistic, what expression would they use to estimate the standard deviation of the sampling distribution of the difference in proportions, $\hat{p}_A - \hat{p}_L$?
- (a) $\sqrt{\frac{12^2}{60} + \frac{17^2}{60}}$
 - (b) $\sqrt{\frac{12}{\sqrt{60}} + \frac{17}{\sqrt{60}}}$
 - (c) $\sqrt{\frac{(0.28)(0.72)}{60} + \frac{(0.2)(0.8)}{60}}$
 - (d) $\sqrt{\frac{(0.24)(0.76)}{60} + \frac{(0.24)(0.76)}{60}}$
 - (e) $\sqrt{\frac{(0.28)(0.72)}{60}} + \sqrt{\frac{(0.2)(0.8)}{60}}$
7. Janice and Linda's test statistic is 1.07. Which of the following is closest to the appropriate P -value for the test?
- (a) 0.0446
 - (b) 0.0892
 - (c) 0.1423
 - (d) 0.1449
 - (e) 0.2846
8. Which of the following best describes what it would mean if Janice and Linda's test resulted in a Type I error?
- (a) Concluding that there is a difference in the proportion of rainy days in the two cities when there is no difference.
 - (b) Concluding that there is no difference in the proportion of rainy days in the two cities when there is a difference.
 - (c) Choosing the wrong test procedure, such as using a z -test instead of a t -test.
 - (d) Accepting the alternative hypothesis instead of rejecting the null hypothesis.
 - (e) Accepting the null hypothesis instead of rejecting the alternative hypothesis.

Part 2: Free Response

Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

9. Nicotine patches are often used to help smokers quit. Does giving medicine to fight depression also help? A randomized double-blind experiment assigned 244 smokers to receive nicotine patches and another 245 to receive both a patch and the antidepressant drug bupropion. After a year, 40 subjects in the nicotine patch group had abstained from smoking, as had 87 in the patch-plus-drug group.
- (a) Construct and interpret a 99% confidence interval for the difference in the proportion of smokers who abstain when using bupropion and a nicotine patch and the proportion who abstain when using only a patch.

- (b) Based only on this interval, do you think that the difference in proportion of abstaining smokers is significant? Justify your answer.

10. Researchers studying the acquisition of pronunciation often compare measurements made on the recorded speech of adults and children. One variable of interest is called "voice onset time" (VOT), the length of time between the release of a consonant sound (such as "p") and the beginning of an immediately following vowel (such as the "a" in "pat"). For speakers of English, this short time lag can be heard as a period of breathiness between the consonant and the vowel.

Here are the results for some randomly-selected 4-year-old children and adults asked to pronounce the word "pat." VOT is measured in milliseconds and can be either positive or negative.

	n	\bar{x}	s
Children	10	60.67	39.89
Adults	20	88.17	24.74

You are interested in whether there is a difference in the VOT of adults and children, so you plan to test $H_0: \mu_A - \mu_C = 0$ against $H_a: \mu_A - \mu_C \neq 0$, where μ_A and μ_C are the population mean VOT for adults and children, respectively.

- (a) What additional information would you need to confirm that the conditions for this test have been met?
- (b) Assuming the conditions have been met, calculate the test statistic and P -value for this test.
- (c) Interpret the P -value in the context of this study, and draw the appropriate conclusion at the $\alpha = 0.05$ level.
- (d) Given your conclusion in part (c), which type of error, Type I or Type II, is it possible to make? Describe that error in the context of this study.

Test 10A

Part 1

1. c The “pairs” are the twelve sets of 10-minute time intervals—one at each intersection. The parameter of interest is the mean difference between the number of cars at each intersection.
2. c The null (“no effect”) hypothesis is that the means are the same, and “We wish to test if the two varieties are significantly different...” calls for a two-tailed test.
3. e Since the P -value (0.0791) is larger than α (0.05), we do not have sufficient evidence against the null, so we cannot reject it. A test of significance only provides evidence against the null hypothesis—it does not provide for the alternative if the P -value is small or for the null if the P -value is large.
4. b The critical t for 9 degrees of freedom is 2.262. The formula for the confidence interval is
$$(\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}.$$
5. e Since both sample sizes are less than 15, t -procedures are not recommended in the presence of skew or outliers. This is especially true when the two sample sizes are quite different.
6. d For tests of significance, use the combined estimate $\hat{p}_c = \frac{x_1 + x_2}{n_1 + n_2}$ in the formula
$$\sqrt{\frac{\hat{p}_c(1 - \hat{p}_c)}{n_1} + \frac{\hat{p}_c(1 - \hat{p}_c)}{n_2}}.$$
7. e Percentile for $z = 1.07$ is 0.8577. Area above $z = 1.07$ is $1 - 0.8577 = 0.1423$. Doubled for two-tailed test is 0.2846.
8. a Type I error is rejecting H_0 when it's true, so in this context, it's concluding that there is a difference in the proportion of rainy days when there is no difference.

Part 2

9. (a) State: We wish to estimate, with 99% confidence, the difference $p_B - p_C$, where p_B and p_C are, respectively, the proportion of subjects who abstain from smoking for a year while being treated with bupropion and nicotine patches or with patches alone (control). Plan: We should use a 2-sample z-interval for $p_B - p_C$. Conditions: *Random*: Subjects were randomly assigned to one of the two treatment groups. *10%*: Since no sampling took place, the 10% restriction does not apply. *Normal/Large Sample*: Number of successes and failures in the two groups are 87, 158, 40, and 204, all of which are at least 10. Do: The critical z for 99% confidence is 2.576, and $\hat{p}_B = \frac{87}{245} = 0.355$; $\hat{p}_C = \frac{40}{244} = 0.164$. The interval is

$$(0.355 - 0.164) \pm 2.576 \left(\sqrt{\frac{(0.355)(0.645)}{245} + \frac{(0.164)(0.836)}{244}} \right) = 0.191 \pm 0.100, \text{ or } (0.091, 0.291). \text{ Conclude:$$

We are 99% confident that the interval from 0.091 to 0.291 captures the true difference in the proportion of smokers using bupropion and nicotine patches who abstain for a year *versus* those using only patches who abstain for a year. (b) Since 0 is not within the 99% confidence interval of plausible values for the difference in proportion of patch-plus-bupropion abstainers and patch-only abstainers, this interval provides convincing evidence that there is a significant difference. A two-tailed test of significance would reject the null hypothesis that $p_B - p_C = 0$ at the $\alpha = 0.01$ level.

10. (a) Since the subjects were randomly selected, the random condition is satisfied. We can certainly assume that the populations from which these subjects were selected are large enough to satisfy the 10% rule. Of greater concern are the small sample sizes: we need to know that there were no outliers in the samples, that the adult sample was not strongly skewed, and that the children sample is not skewed at all, since this sample is particularly small.

$$(b) t = \frac{(88.17 - (60.67)) - 0}{\sqrt{\frac{24.74^2}{20} + \frac{39.89^2}{10}}} = \frac{27.5}{13.77} = 1.997.$$

Using table B and conservative $df = 9$, the two tailed P -value is between 0.05 and 0.10. Using the calculator and $df = 12.57$, P -value = 0.068. (c) If there is no difference between the mean VOT of adults and children, the probability of obtaining a sample mean difference this far or farther from 0 is 0.068. We fail to reject H_0 . We do not have convincing evidence that there is a difference in the mean VOT of adults and children. (d) It's possible we made a Type II error, which would be to conclude that there is no difference in mean VOT between adults and children when there is a difference.