



Redesigned SAT Math Strategy Packet

Use this guide as a quick-reference for the most important concepts and strategies on the Redesigned SAT's mathematics sections.

Key Strategy #1: *Plugging in Numbers*

Use whenever...a word problem's language refers to quantities with variables, as in "p percent", "h hours", etc.

To incentivize bulk purchases of its textbooks, a textbook publisher sells the first 300 textbooks a school purchases at a price of d dollars for each book, and offers a discount of c dollars off the price of each textbook for every textbook purchased after the first 300. If a school purchases a total of t textbooks, which of the following gives the final price of all textbooks purchased, assuming $t > 300$?

- A) $300d + (t - 300)(d - c)$
- B) $300d + (300 - t)(d - c)$
- C) $300d + c(t - 300)$
- D) $300d + c(300 - t)$

Once we see the language in the problem, which refers to quantities by using variables, we know we can Plug in Numbers.

Step 1: Assign numbers for variables. Don't violate any conditions set in the problem (ex: don't use even numbers if it tells you to use odd ones), and pick numbers that are easy for you to use.

$d = 30$; this is the original price charged per book, a nice easy number
 $c = 10$; this is the discount off the original (all books after the 300th cost \$20)
 $t = 500$; this is the total number of books, which must be greater than 300

Step 2: Determine the question and solve the problem with your numbers. The question asks us to solve for the final price for all books. Don't use the variables to solve — just the numbers!

Each of the first 300 books costs \$30.

$$\$30(300) = \$9,000$$

Since we chose 500 as our total number of books, there are 200 left. Each of these will be priced at the discounted amount, \$20, as we chose \$10 to be the value of the discount.

$$\$20(200) = \$4,000$$

The total price of all books purchased is therefore $\$9,000 + \$4,000 = \mathbf{\$13,000}$.

Step 3: Plug the values you set for each variable back into the answer choices until one of them gives you the answer from Step 2.

A) $300(30) + (500 - 300)(30 - 10) = 9,000 + (200)(20) = 9,000 + 4,000 = \mathbf{13,000}$

B) $300(30) + (300 - 500)(30 - 10) = 9,000 + (-200)(20) = 9,000 - 4,000 = 5,000$

C) $300(30) + 10(500 - 300) = 9,000 + 10(200) = 9,000 + 2,000 = 11,000$

D) $300d + c(300 - t) = 300(30) + 10(300 - 500) = 9,000 + 10(-200) = 7,000$

Choice A is correct.

Key Strategy #2: Interpreting Tables

Use whenever...you are dealing with a two-way table, which features row and column totals.

| | Household Location | | |
|-------------------|--------------------|-------|-------|
| Type of Generator | Suburban | Rural | Total |
| Natural Gas | 35 | 11 | 46 |
| Gasoline | 16 | 64 | 80 |
| Total | 51 | 75 | 126 |

A random survey of 51 suburban households and 75 rural households asked homeowners which type of generator they owned. The responses of those who owned generators are summarized in the table above.

Of the respondents who owned a generator, which of the following is closest to the percent of suburban homeowners who owned a natural gas generator?

- A) 76%
- B) 69%
- C) 47%
- D) 28%

The key here is to determine which column total is the most relevant. This involves paying careful attention to the language of the problem, particularly the words following "percent" or "fraction". When you see "percent of" or "fraction of", the category following the phrase will be the relevant total.

Need to find: "**Percent of**" suburban homeowners who owned a natural gas generator.

Therefore, the relevant total is the total number of suburban homeowners, or 51.

$$\frac{35}{51} \times 100 = 68.7\% \approx 69\%$$

Therefore, Choice B is correct.

Key Strategy #3: Know Your Algebraic Manipulations

Use whenever...a problem asks you to solve for one or multiple variables in terms of other variables or real numbers.

Solving for a variable that is part of two different terms. In general, group the like terms and factor out the variable you're solving for. Here, we solve for a in $ab - 5 = 4a$.

$$ab - 5 = 4a \quad \text{The variable } a \text{ is in two expressions}$$

$$ab - 4a = 5 \quad \text{Get both terms that contain } a \text{ on the same side by subtraction and addition}$$

$$a(b - 4) = 5 \quad \text{Factor an } a \text{ out of both sides}$$

$$a = \frac{5}{b-4} \quad \text{Divide to solve for } a$$

Solving in terms of multiple variables. In general, cross multiply when you have a fraction equal to an expression or to another fraction. Here, we solve for $\frac{a}{b}$ given the equation $\frac{2a}{5} = 7b$.

$$\frac{2a}{5} = 7b \quad \text{We need the } b \text{ under the } a. \text{ First, linearize by multiplying both sides by 5}$$

$$2a = 35b \quad \text{Now we can get the } b \text{ under the } a \text{ by dividing by } b \text{ on both sides}$$

$$\frac{2a}{b} = 35 \quad \text{Now we must isolate } \frac{a}{b} \text{ by dividing by 2 on both sides}$$

$$\frac{a}{b} = \frac{35}{2} \quad \text{Solution in terms of two variables}$$

Dealing with radical expressions. In general, when a variable is under a radical, square both sides of the equation to eliminate the root sign. Here, we solve for b in $\sqrt{2-b} = 2\sqrt{b}$.

$$(\sqrt{2-b})^2 = (2\sqrt{b})^2 \quad \text{Square both sides to eliminate the radical signs}$$

$$2 - b = 4b \quad \text{Combine like terms by adding } b \text{ to both sides}$$

$$2 = 5b \quad \text{Solve for } b \text{ by division}$$

$$b = \frac{2}{5} \quad \text{Solution}$$

Key Strategy #4: Get Comfortable with Systems of Equations

Use whenever...you see the term "system of equations" or are presented with two or more equations that have multiple common unknowns.

When you've got two equations "stacked", you can add or subtract them just like you would numbers. To solve for one of the variables, you must eliminate the other.

$$\begin{aligned}4m + 7n &= 16 \\2m - 3n &= 5\end{aligned}$$

If (m, n) is the solution to the system of equations above, what is the value of n ?

Step 1: Decide which variable to get rid of. This is the one that you're *not* trying to solve for. Make the coefficient in front of the variables you want to eliminate the same in both equations by multiplying by whatever is necessary. We multiply the bottom equation by 2 because we want to solve for n by eliminating m .

$$\begin{aligned}4m + 7n &= 16 \text{ ----> } 4m + 7n = 16 \\2 [2m - 3n &= 5] \text{ ----> } 4m - 6n = 10\end{aligned}$$

Step 2: Subtract the equations, noting that the negative subtraction sign will distribute to any negatives in the bottom equation, turning them positive.

$$\begin{array}{r}4m + 7n = 16 \\- (4m - 6n = 10) \\ \hline13n = 6\end{array}$$

$$n = \frac{13}{6}$$

When you have multiple variables and one is equal to an expression in terms of the other, you can use substitution.

$$\begin{aligned}x &= 2c + y \\y &= x - 4\end{aligned}$$

The system of equations above is true for all values (x, y) , and c is a constant. What is the value of c ?

Note that the equations share variables, and that the second relates y in terms of x . We can substitute the quantity $x - 4$ for y in the first equation.

$$\begin{aligned}x &= 2c + y \text{ ----> } x = 2c + x - 4 \\2c - 4 &= 0 \text{ ----> } 2c = 4 \text{ ----> } c = 2\end{aligned}$$

Key Strategy #5: Understand Slope and y-Intercept Terms

Use whenever...you are asked to interpret the slope or y-intercept terms in a linear equation. Questions that ask for these always use language like “what is the meaning” or ask how a certain value is “interpreted”.

The slope term gives you the change in the y variable for every 1-unit change in the x variable.

The price, p , of an item in dollars is modeled by the equation $p = 2.4t + 5$, where t represents the time in years since the item goes on sale. Which of the following explains the 2.4 in the equation?

- A) Every 2.4 years, the price of the item will increase by \$1.00.
- B) With each additional year, the price of the item is multiplied by 2.4.
- C) For each additional year, the price of the item increases by \$2.40.
- D) The price of the item can be found by multiplying the number of years by 2.4.

This is a linear equation, because it is in the form $y = mx + b$, where the price, p , is our y variable, and the time in years, t , is our x variable. Therefore, the 2.4 is the slope term (the m in $y = mx + b$), and indicates that the y variable changes by 2.4 units for every 1-unit change in the x variable. In the context of the problem, this means the price increases by \$2.40 for each additional (1) year that goes by. Choice C is correct, because it is the only answer that phrases this interpretation properly.

The y-intercept term gives you the value of the y variable when x is equal to 0.

The price, p , of an item in dollars is modeled by the equation $p = 2.4t + 5$, where t represents the time in years since the item goes on sale. Which of the following explains the 5 in the equation?

- A) The initial price of the item is \$5.00.
- B) The price of the item increases by \$5.00 each year.
- C) For every 5 years that go by, the price of the item increases by \$1.00.
- D) An item sold after the first year will cost \$5.00.

This is the same linear equation as before. Clearly, 5 is the b term in $y = mx + b$, so it represents the y-intercept, which tells us the y-value (in this case, p , the item's price) when the x-value (in this case, t , the number of years since the item is put on sale) is equal to 0. If the number of years is 0, this means that \$5.00 is the price 0 years after it is put on sale; in other words, it's the initial price of the item. Choice A explains this correctly.

Sometimes, you'll have to rearrange a linear equation to put it into $y = mx + b$ form.

$$10y + 3x = 7 \text{ ----> } y = -\frac{3}{10}x + \frac{7}{10}$$

Here, for example, the slope term here indicates that there is a $\frac{3}{10}$ decrease in the y-variable for every 1-unit change in the x-variable.

Key Strategy #6: With Percents, Translate Words to Math and Use Shortcuts for Calculating

Use whenever...you are dealing with a word problem that involves percents.

To find a percent “of” a quantity, convert the percent to decimal form by moving its decimal point two places to the left, and then multiply by the quantity. Translate any other operations as normal.

In 2013, a business earned 40 percent of its \$200,000 revenue from software sales. In 2014, the business earned \$10,000 more than 50 percent of its \$300,000 revenue from software sales. How much revenue in total did the business earn from software sales over the two years?

- A) \$100,000
- B) \$240,000
- C) \$300,000
- D) \$360,000

Translate the words to math, using decimal equivalents of percents.

“40 percent of \$200,000” in 2013 means $0.40 \times 200,000 = 80,000$

“\$10,000 more than 50 percent of \$300,000” means $10,000 + (0.50 \times 300,000) = 160,000$

$$80,000 + 160,000 = 240,000$$

Choice B is correct.

To increase a quantity by a certain percent in a single step, *add* the decimal equivalent of the percent increase to 1, and then multiply by the quantity. To decrease a quantity by a certain percent in a single step, *subtract* the decimal equivalent of the percent decrease from 1, and then multiply by the quantity.

In 2010, there were 450 members of a local organization in Fairmont County. In 2011, the number of members grew by 20 percent. In 2012, the number of members then decreased by 20 percent by the end of the year. How many members were in the local organization by the end of 2012?

- A) 72
- B) 260
- C) 410
- D) 432

Step 1: Increase 450 by 20% by converting 20% to 0.20, adding 1, and multiplying by 450.

$$450(1 + 0.20) = 450(1.20) = 540$$

Step 2: Decrease 540 by 20% by converting 20% to 0.20, subtracting from 1, and multiplying by 540.

$$540(1 - 0.20) = 540(0.80) = 432$$

Choice D is correct.

Key Strategy #7: Master Averages by Using the Formula

Use whenever...you see the word "average", "mean", or "arithmetic mean".

$$\text{Average} = \frac{\text{Sum of Numbers}}{\text{Number of Numbers}}$$

Write this formula down immediately when you see these words in the problem text. Use them as a template for summarizing the information and setting up an equation that is easily solved.

If a is the average of b and c and x is the average of y and z , what is the average of b , c , y , and z in terms of x and y ?

- A) $\frac{a+x}{4}$
- B) $\frac{a+x}{2}$
- C) $a + x$
- D) $2a + 2x$

Step 1: Write down the average formula. For every statement that includes the word "average", write a new formula and put the information in the correct places.

$$\text{Average} = \frac{\text{Sum of Numbers}}{\text{Number of Numbers}}$$

$$a = \frac{b+c}{2} \quad x = \frac{y+z}{2}$$

Step 2: Usually, you must manipulate by cross-multiplying.

$$2a = b + c \quad 2x = y + z$$

Step 3: Determine what the problem wants. We're asked for the average of b , c , y , and z in terms of x and y . Again, we write the average formula and plug in the numbers in the correct places. You may need to use a substitution in more advanced problems.

$$\text{Average} = \frac{(b + c) + (y + z)}{4}, \text{ and by substitution, Average} = \frac{2a + 2x}{4} = \frac{a + x}{2}$$

Choice B is correct.

Key Strategy #8: Use Key Coordinate Geometry Concepts

Use whenever...you are dealing with lines or polynomials on the xy -plane

Perpendicular lines have negative reciprocal slopes. Given the slope of one of the lines, "flip" the slope and negate it to find the slope of a line perpendicular to it. The y -intercept does not matter. Parallel lines have the same slope. Always rewrite linear equations in $y = mx + b$ form.

$$y = -3x + 5 \quad \text{and} \quad y = \frac{1}{3}x - 2 \text{ are perpendicular.}$$

Whenever you are given a graph's equation and a point on that graph, that point's coordinates can be plugged in for x and y in the equation to solve for unknowns.

If the problem refers to the " x -intercept" (where the line/curve crosses the x -axis), fill in 0 for y .
If the problem refers to the " y -intercept" (where the line/curve crosses the y -axis), fill in 0 for x .

At the point where the line $y = 2c + 8$ crosses the x -axis, $x = -4$. What is the value of c ?

The graph crosses the x -axis at the x -intercept, so y must equal 0. We plug in 0 for y and -4 for x to solve.

$$0 = 2c(-4) + 8 \rightarrow 0 = -8c + 8 \rightarrow 8c = 8 \rightarrow c = 1$$

To determine where the graphs of two lines/curves intersect, set the expressions that are equal to y equal to each other.

The graphs of $y = 2x$ and $y = x^2$ intersect at points (a, b) and (c, d) . What is the value of $a + c$?

Set the expressions that are equal to y equal to each other and solve for x .

$$\begin{aligned} 2x &= x^2 \\ x^2 - 2x &= 0 \\ x(x - 2) &= 0 \\ x &= 0 \text{ and } x = 2 \end{aligned}$$

Since a and c are simply the x -coordinates of the intersection points, they are the values of x we just solved for. The solution is therefore $0 + 2 = 2$.

When given points in function notation, remember that $f(x) = y$. The number in parentheses is the x -coordinate of the point, and the number the expression is equal to is the y -coordinate.

$$f(5) = 3 \text{ indicates the point } (5, 3) \text{ is on the function } f$$

Remember that slope can be phrased as part of an equation, not just as an expression, and that when you are given the slope but are missing points, you can substitute the slope for the $\frac{\Delta y}{\Delta x}$ expression.

$$\frac{\text{Rise}}{\text{Run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

On the xy -plane, Ronald creates a scale drawing of a ramp he will build. He knows the slope of the ramp must be a 20% grade (that is, for every 2 feet of vertical rise, the ramp will measure 10 feet in length along the ground). On the drawing, Ronald places one end of the ramp at $(4, 5)$, and the other at $(p, 8)$. What is the value of p ?

Since slope is involved and a point is unknown, use the slope equation. According to the problem, the slope is $\frac{2}{10}$, which can be substituted for $\frac{\Delta y}{\Delta x}$. The points can then be plugged in on the right side.

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \text{ ----> } \frac{2}{10} = \frac{8 - 5}{p - 4} \text{ ----> } \frac{2}{10} = \frac{3}{p - 4} \text{ ----> } 2(p - 4) = 30 \text{ ----> } 2p - 8 = 30$$

$$2p = 38 \text{ ----> } p = 19$$

Key Strategy #9: Look for Ways to Use Exponent Rules

Use whenever...you are dealing with exponents or roots that must be phrased as powers.

Know your rules. Cold.

$$x^{a+b} \text{ comes from } (x^a)(x^b)$$

$$x^{a-b} \text{ comes from } \frac{x^a}{x^b}$$

$$x^{ab} \text{ comes from } (x^a)^b$$

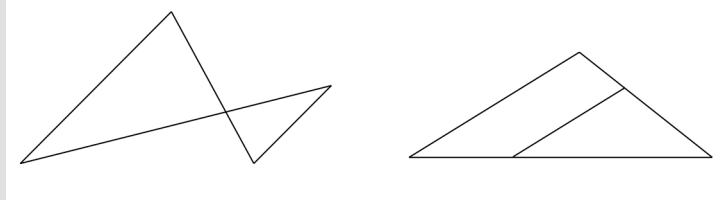
$$x^{\frac{a}{b}} = \sqrt[b]{x^a}$$

When a problem mixes exponents and roots, convert the root expression to exponential form

$$\frac{\sqrt[5]{x^4}}{x^{\frac{2}{3}}} = \frac{x^{\frac{4}{5}}}{x^{\frac{2}{3}}} = x^{\frac{4}{5} - \frac{2}{3}} = x^{\frac{2}{15}}$$

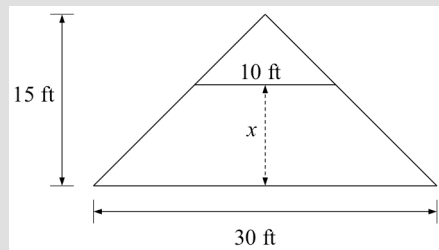
Key Strategy #10: Recognize Similar Triangles

Use whenever...you see two triangles that are oriented in one of the following ways.



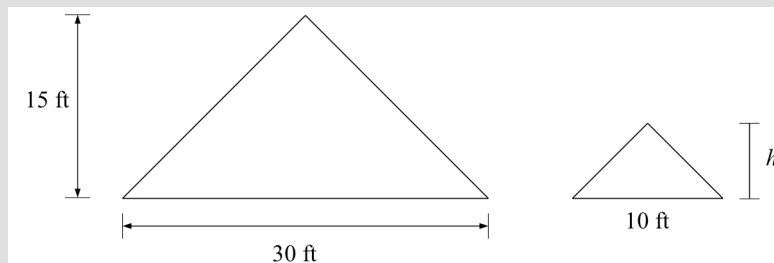
Triangles with parallel bases sharing a vertical angle (left) and the "triangle-in-a-triangle" with parallel sides are two common similar triangle setups.

Similar triangles have three equal angles and proportional sides. When you see similar triangles and need to solve for an unknown side length, set up a proportion, placing the measures of corresponding sides over one another. Sometimes, it helps to draw the triangles separately.



The figure above shows an isosceles triangular roof with a 30-foot base and 15-foot height. A 10-foot crossbeam is to be installed a distance x feet from the base of the roof for support. What is the value of x ?

We can't solve directly for x , because it is not itself a side of any triangle. We can separate the triangles though, and set up a proportion to solve for the height of the small triangle (shown below as h). Then, we'll subtract this from the total height to solve for x , which is the leftover height above the base of the roof.



$$\frac{15}{h} = \frac{30}{10} \rightarrow 30h = 150 \rightarrow h = 5$$

Since $h = 5$ and the roof is 15 feet high, the distance $x = 10$ feet.

Key Strategy #11: Understand Linear and Exponential Functions

Use whenever...you are asked what kind of model or function would best describe or fit a data set.

Linear functions have a constant rate of change (slope): their y -value increases or decreases by the same amount for equivalent changes in x -values. The y -value of the function whose points are shown below decreases by 3 for every 1-unit increase in x (slope = -3).

$$y = -3x + 11$$

| | | | | | |
|-----|---|---|---|----|----|
| x | 1 | 2 | 3 | 4 | 5 |
| y | 8 | 5 | 2 | -1 | -4 |

Exponential functions increase or decrease by a constant percent or fraction of the previous value. The y -value of the function whose points are shown below decreases by 80% of its previous value for each 1-unit increase in x .

$$y = 1000(0.2)^x$$

| | | | | | |
|-----|-----|----|---|-----|------|
| x | 1 | 2 | 3 | 4 | 5 |
| y | 200 | 40 | 8 | 1.6 | 0.32 |

Note that to make an exponential equation of the form $y = a(b)^x$ increase by p percent, add the decimal equivalent of p to 1 and make that quantity b . To make it decrease by p percent, subtract the decimal equivalent of p from 1 and make that quantity b . It's clear that in the exponential equation above, the function's value decreases by 80%, because $b = 0.2$, which is the same as $1 - 0.8$.

Key Strategy #12: Understand Function Notation for Graph Shifts

Use whenever...you are asked how the graph of a function will change when given a manipulation of $f(x)$.

Given the original function $f(x)$...

$f(x + k)$ shifts the graph k units left; $f(x - k)$ shifts the graph k units right

$f(x) + k$ shifts the graph k units up; $f(x) - k$ shifts the graph k units down

$k(f(x))$ stretches the graph: if $k > 1$, the graph narrows; if $0 < k < 1$, the graph widens

Key Strategy #13: Use Proportions to Perform Unit Conversions

Use whenever...you are given a word problem that asks you to express a quantity in one unit in terms of a different unit.

The *nautical league*, an ancient unit of measurement for seafarers, is equivalent to 3.54 miles. The *fathom*, another unit of nautical measurement, is equal to 2 yards. If each yard is equal to 3 feet, how many fathoms are in 2 leagues? (1 mile = 5,280 feet)

Step 1: Start by listing your conversion factors, which tell you how many of one unit are equal to another unit.

$$\begin{aligned}1 \text{ league} &= 3.54 \text{ miles} \\1 \text{ fathom} &= 2 \text{ yards} \\1 \text{ yard} &= 3 \text{ feet} \\1 \text{ mile} &= 5,280 \text{ feet}\end{aligned}$$

Step 2: Keep track of the end goal. We want to go from leagues to fathoms, but since we don't have a direct conversion, we need to go through miles, feet, and yards. Start with the final given value you need to convert: 2 leagues. We can convert these to miles since we have a direct equivalence.

$$\frac{1 \text{ league}}{3.54 \text{ mi}} = \frac{2 \text{ leagues}}{x \text{ mi}} \text{ ----> } x = 6.9 \text{ mi}$$

Step 3: Now, move through the other units, simply using proportions each time with each of our conversion factors. We can go from miles to feet, then feet to yards, then yards to fathoms. For consistency, use the left side for your conversion factors, and the right side for the value you're converting.

$$\frac{1 \text{ mi}}{5,280 \text{ ft}} = \frac{6.9 \text{ mi}}{x \text{ ft}} \text{ ----> } x = 36,432 \text{ ft}$$

$$\frac{3 \text{ ft}}{1 \text{ yd}} = \frac{36,432 \text{ ft}}{x \text{ yd}} \text{ ----> } 3x = 36,432 \text{ ----> } x = 12,144 \text{ yd}$$

$$\frac{2 \text{ yd}}{1 \text{ fathom}} = \frac{12,144 \text{ yd}}{x \text{ fathoms}} \text{ ----> } 2x = 12,144 \text{ ----> } x = 6,072 \text{ fathoms}$$

You can also use units to help determine which calculation to perform in simpler but still challenging circumstances. For example, let's say you're told you can buy 10 notebooks for d dollars, and want to know how much money, in terms of d , it would cost to buy 17 notebooks. You can build a proportion, or can choose to set up a multiplication like this, noting that the "notebooks" unit cancels, and that you're left with dollars:

$$\frac{d \$}{10 \text{ notebooks}} \times 17 \text{ notebooks} = \frac{17d}{10}$$

Key Strategy #14: Know the Easy Ways Out for Quadratics or Cubics

Use whenever...you are given a quadratic equation and need to manipulate it.

Remember that factors give you roots, and roots give you points on a function.

| | |
|------------------------|---------------|
| $x^2 + 2x - 8$ | Expression |
| $(x - 2)(x + 4)$ | Factors |
| $(2, 0)$ and $(-4, 0)$ | Zeros (Roots) |

If $(x + 2)$ is one factor of the quadratic expression $3x^2 - kx - 16$, what is the value of k ?

If $(x + 2)$ is a factor of the quadratic expression, it means that the function $y = 3x^2 - kx - 16$ goes through the point $(-2, 0)$ on the graph. We can therefore simply plug in -2 for x and 0 for y to solve for k .

$$\begin{aligned}0 &= 3(-2)^2 - k(-2) - 16 \\0 &= 12 + 2k - 16 \\0 &= 2k - 4 \rightarrow k = 2\end{aligned}$$

Recognize *vertex form*, which gives you the vertex, or turning point, of the parabola. This must be the maximum for up-facing parabolas and the minimum for down-facing parabolas. Reverse the sign of the number in the parentheses with the x for the x -coordinate of the vertex. The constant term outside the parentheses is the y -coordinate of the vertex.

$$y = (x + 1)^2 - 16$$

The vertex is $(-1, -16)$, so $y = -16$ is the minimum value of the function. Furthermore, the axis of symmetry is $x = -1$, because this line passes through the turning point. Manipulate by FOIL to convert to standard form if necessary. Standard form can then be factored to find zeros/roots.

$$\begin{aligned}y &= (x + 1)(x + 1) - 16 \rightarrow y = x^2 + 2x + 1 - 16 \\y &= x^2 + 2x - 15 \\y &= (x - 3)(x + 5) \\(3, 0) &\text{ and } (-5, 0)\end{aligned}$$

Note that the constant term in standard form gives the y -intercept, which is $(0, -15)$ in this case.

Use shortcuts to find the sum and product of the roots and the axis of symmetry when necessary.

$$y = ax^2 + bx + c$$

$$\text{Sum of roots: } -\frac{b}{a} \quad \text{Product of Roots: } \frac{c}{a} \quad \text{Axis of Symmetry: } x = -\frac{b}{2a}$$